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DISCRETE POPULATION GROWTH MODEL WITH CONSTANT FORCED REPRODUCTION AND CONSTANT HARVESTING

Rajat Kaushik*, Sandeep Kumar, Alok Kumar

* Indian Institute of technology Roorkee, Roorkee-247667, Uttrakhnad (India)
Indian Airforce Academy, Dundigal, Hyderabad-500043, Telangana (India)
Dev Sanskriti Vishwvidhyalay, Haridwar-249401, Uttrakhand (India)

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ABSTRACT

A generalization of discrete logistic growth model is considered. Constant harvesting for human consumption of species is taken into account. At the same time, forced reproduction of the same species is also considered. The constant cycle of forced pregnancy and birth creates a huge surplus of species. A general framework of fixed point's stability is carried out taking all possible conditions of harvesting and reproduction. For evenness, limited harvesting as well as balanced forced reproduction is advisable.

KEYWORDS: Logistic growth; stability, chaos, period doubling, fixed points.

INTRODUCTION

As far as theoretical and practical importance is concern, population growth of any species can be indicated by logistic growth model which provides most basic fundamental foundation of population dynamics. The classical discrete logistic growth model can be shown by-

$$y_{n+1} = \mu y_n(1 - y_n)$$

Where μ is a parameter in R_+ and y_n be considered in [0, 1].

Due to human consumption and need, harvesting of the different species begins. Growing need for human consumption gives birth to the heavy harvesting of species and cattle. Now since natural births are limited in nature, therefore impact of heavy harvesting results in the extinction of the species. In order to maintain the number of species, forced reproduction techniques are used. In this process, the constant cycle of forced pregnancy and birth creates a huge surplus of species which possibly maintain the huge harvesting effect.

MODEL EQUATIONS AND ANALYSIS

We consider the simplest equation with constant forced reproduction and constant harvesting as-

$$s_{n+1} = \alpha s_n(1 - s_n) + \beta - \gamma \quad (1)$$

Where-
$\beta =$ Constant forced reproduction
$\gamma =$ constant harvesting
Now three cases arises-
Case 1: $\beta = \gamma$
First we consider the case where harvesting is equal to the forced reproduction. Then equation 1 reduces to-

$$s_{n+1} = \alpha s_n(1 - s_n)$$

This simple model has 2 fixed points-

$$s_0^* = 0 \qquad and \qquad s_1^* = 1 - \frac{1}{\alpha}$$

- When $0 < \alpha < 1$ only $s_0^* = 0$ fixed point exist which is stable.
- When $1 < \alpha < 3$, both fixed point exist. s_0^* Is repelling while s_1^* is attracting.
- When $\alpha > 3$ both fixed points are unstable. But at the same time there is period two orbit which is stable for $3 < \alpha < 1 + \sqrt{6}$ and unstable for $1 + \sqrt{6} < \alpha < 4$.

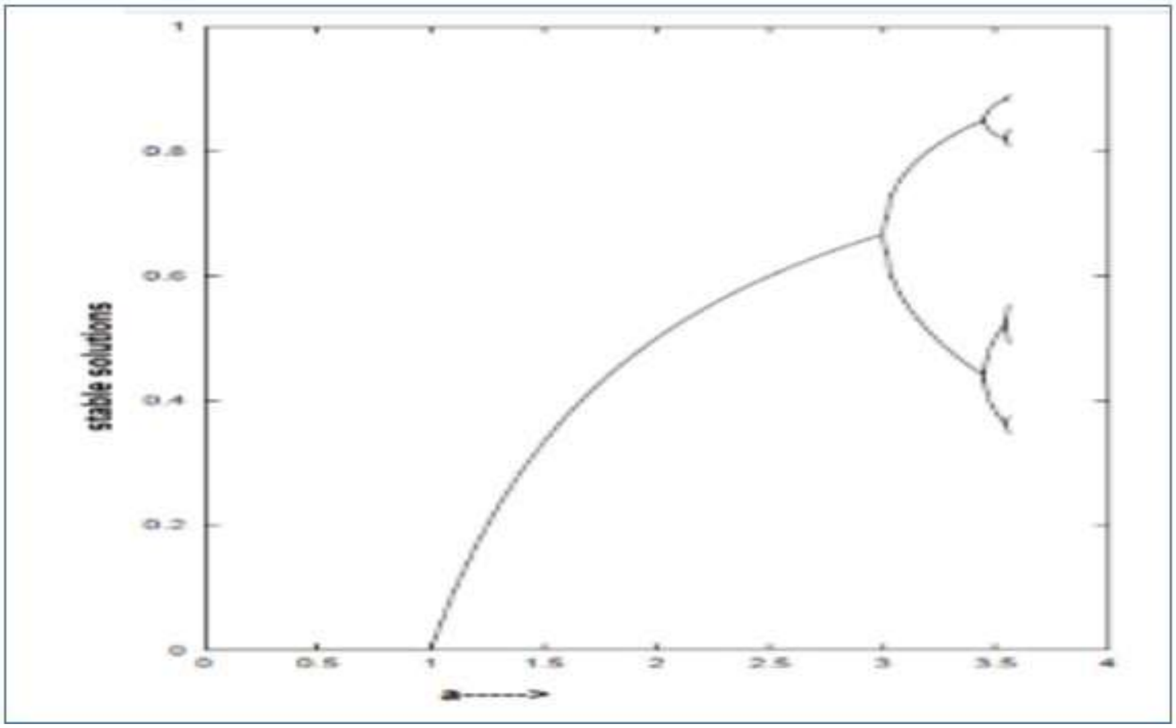

Figure: 1

As **a** move forward, period 4 orbit arises which also changes type after a critical value of **a** and then a period 8 cycle is born. This process continues to give a period doubling cascade and this period doubling route to chaos.

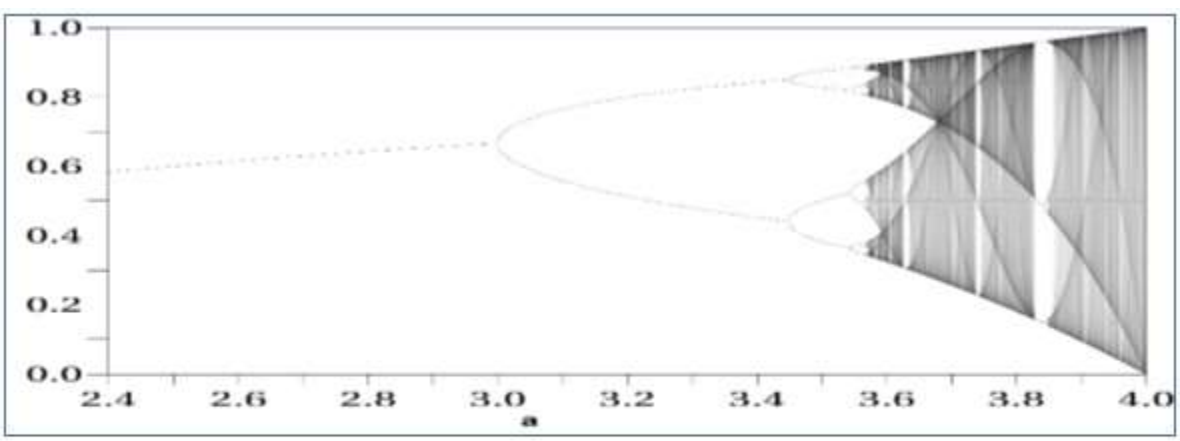

Figure: 2

Case 2: $\beta > \gamma$:
This is the case when forced reproduction is more than harvesting.
Let $\beta - \gamma = \tau$ then equation 1 reduces to-

$$s_{n+1} = \alpha s_n(1 - s_n) + \tau = f(s_n)$$

Fixed Points: We have only one positive fixed point as-

$$s_1^* = \frac{(\alpha - 1) + \sqrt{(\alpha - 1)^2 + 4\alpha\tau}}{2\alpha}$$

For example if we have $s_{n+1} = s_n(1 - s_n) + 0.5$ then positive fixed point can be shown as-

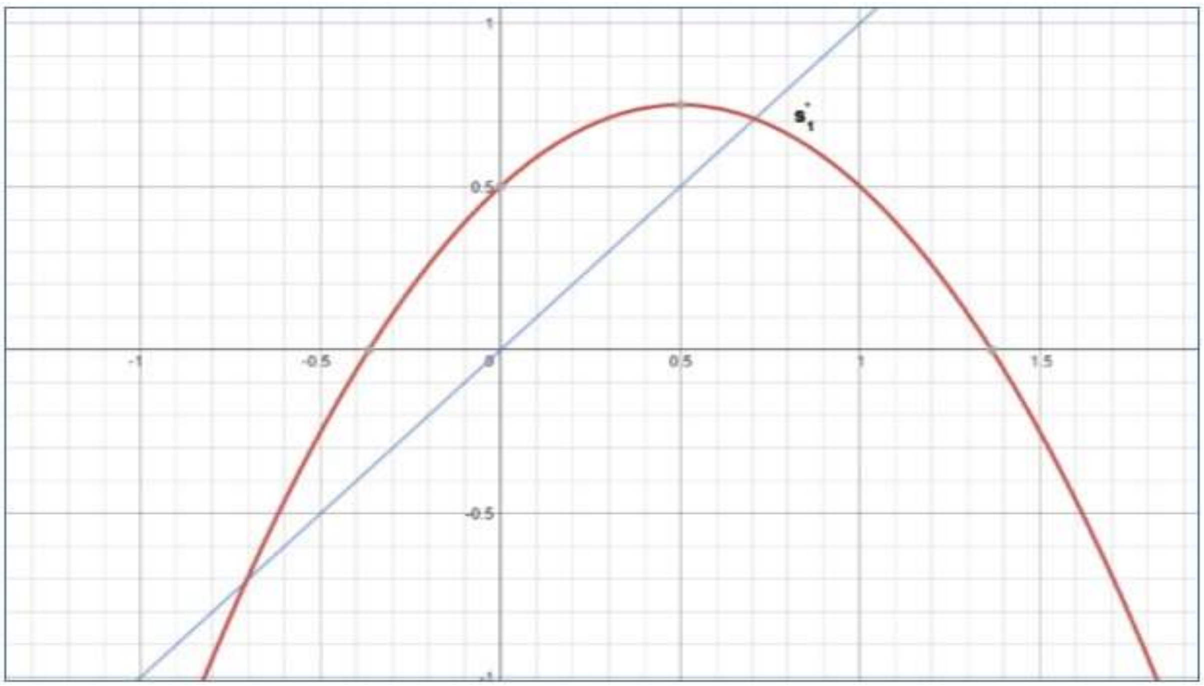

Figure: 3

Stability: Condition for stability is-

$$|f'(s^*)| < 1$$

Hence s_1^* will be stable if-

$$\left|1 - \sqrt{(\alpha - 1)^2 + 4\alpha\tau}\right| < 1$$

$$0 < \sqrt{(\alpha - 1)^2 + 4\alpha\tau} < 2$$

Two periodic fixed points: For two periodic fixed points we must have-

$$s_{n+2} = s_n$$

Hence two periodic fixed points are the roots of the equation-

$$\alpha^3x^4 - 2\alpha^3x^3 + (\alpha^3 + \alpha^2 - 2\tau\alpha^2)x^2 + (2\tau\alpha^2 - \alpha^2 + 1)x + (\tau^2\alpha - \tau - \tau\alpha) = 0$$

Since one period fixed points are obviously two period fixed points. So two period fixed points which are not one period fixed points are the positive roots of the equation-

$$\alpha^2x^2 - (\alpha^2 + \alpha)x + (\alpha + 1 - \tau\alpha) = 0$$

Case 3: $\beta < \gamma$:
This is the case when forced reproduction is less than harvesting.
Let $\beta - \gamma = -\sigma$ then equation 1 reduces to-

$$s_{n+1} = \alpha s_n(1 - s_n) - \sigma = f(s_n)$$

Fixed Points: We have two positive fixed point if-

$$(\alpha - 1)^2 > 4\alpha\sigma \qquad and \qquad \alpha > 1$$

Fixed points are-

$$s_1^* = \frac{(\alpha - 1) + \sqrt{(\alpha - 1)^2 - 4\alpha\sigma}}{2\alpha}$$

$$s_2^* = \frac{(\alpha - 1) - \sqrt{(\alpha - 1)^2 - 4\alpha\sigma}}{2\alpha}$$

But if $\alpha < 1$ then no positive fixed points are obtained.
For example if we have $s_{n+1} = 3s_n(1 - s_n) - 0.1$ then positive fixed points can be shown as-

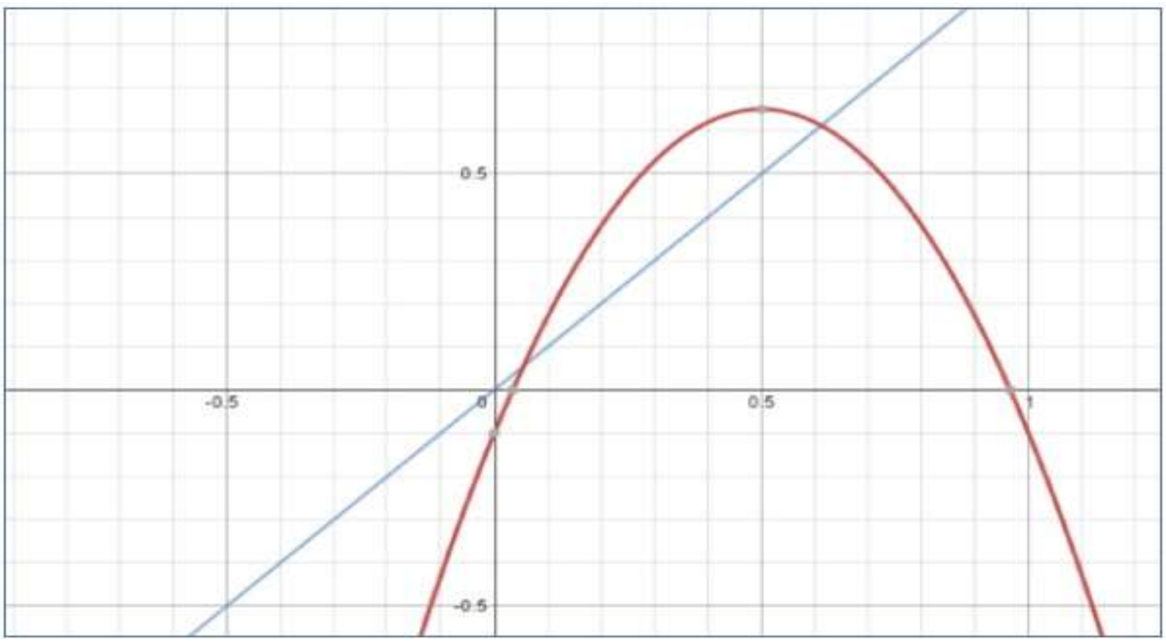

Figure: 4

If we have $s_{n+1} = 0.5s_n(1 - s_n) - 0.05$ then no positive fixed points are obtained as $\alpha < 1$.
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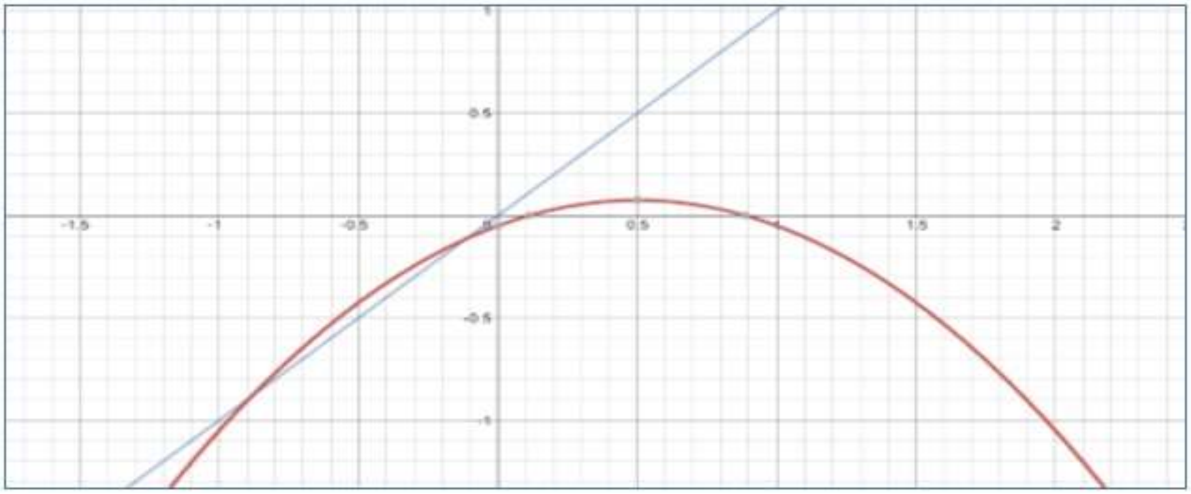

Figure: 5

If we have $s_{n+1} = 0.5s_n(1 - s_n) - 0.5$ then no fixed points are obtained as $(\alpha - 1)^2 < 4\alpha\sigma$.

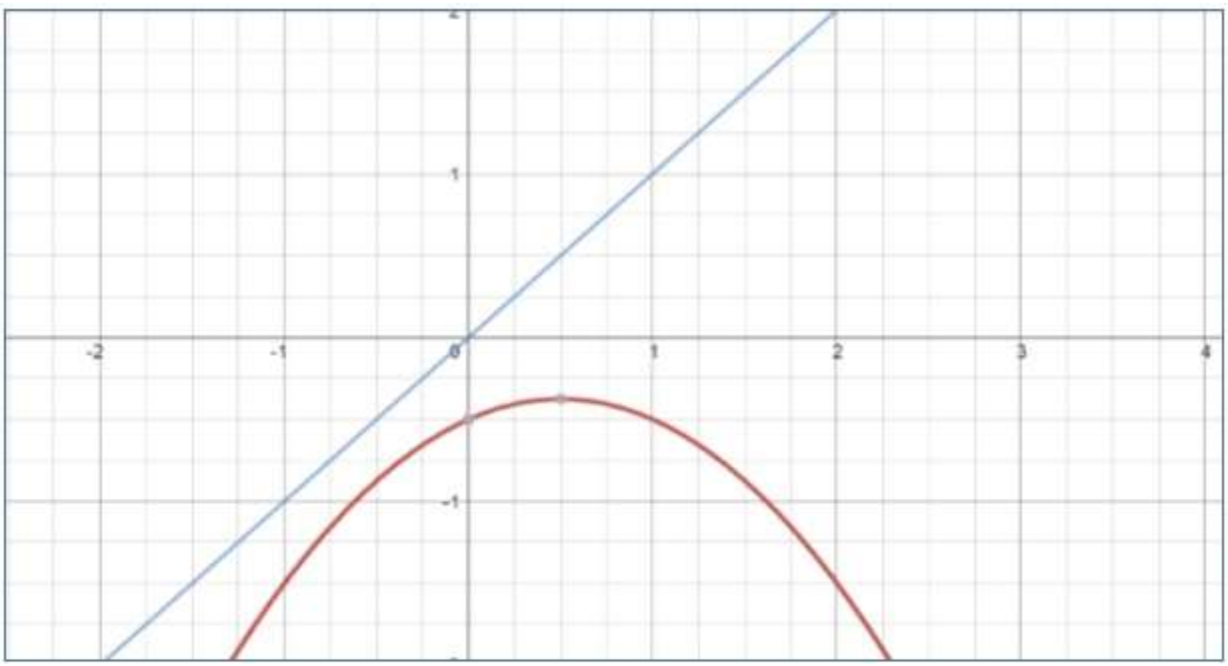

Figure: 6

CONCLUDING REMARKS

In the analysis, it can be easily seen that if harvesting is more than forced reproduction then fixed points are hardly gettable. Also redundant forced reproduction can cause suffering and increment in mortality of the species. Hence a balanced harvesting as well as forced reproduction is advisable for regular process. Stable fixed points are also obtained and their stability analysis is also carried out where needed. Research work can be further modified taking additional mortality due to forced pregnancy or otherwise into account.

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